§3.4

• #14: Find the general solution of the given differential equation. \( 9y'' + 9y' - 4y = 0 \).

**Solution.** The characteristic equation \( 9r^2 + 9r - 4 = 0 \) has roots 
\[
\frac{1}{3}, \quad \frac{-4}{3}.
\]
The general solution of the differential equation is 
\[
y(t) = c_1e^{t/3} + c_2e^{-4t/3}.
\]

#19: Find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior for increasing \( t \).
\[
y'' - 2y' + 5y = 0, \quad y(\pi/2) = 0, \quad y'(\pi/2) = 2.
\]

**Solution.** The characteristic equation \( r^2 - 2r + 5 = 0 \) has roots 
\[
1 + 2i, \quad 1 - 2i
\]
The general solution of the differential equation is
\[
y(t) = c_1 e^t \cos(2t) + c_2 e^t \sin(2t).
\]
\[
y'(t) = c_1 e^t \cos(2t) - 2c_1 e^t \sin(2t) + c_2 e^t \sin(2t) + 2c_2 e^t \cos(2t)
\]
y(\pi/2) = 0 implies that 
\[
-c_1 e^{1/2\pi} = 0;
\]
y'(\pi/2) = 2 implies that 
\[
-c_1 e^{(\pi/2)} - 2c_2 e^{(\pi/2)} = 2.
\]
Solve for \( c_1 \) and \( c_2 \), we have \( c_1 = 0, \ c_2 = -e^{-1/2\pi} \). Thus,
\[
y(t) := -e^{(t-\frac{\pi}{2})} \sin(2t)
\]
y(t) oscillates with increasing amplitude as \( t \) increases.
#23: Consider the initial value problem

\[ 3u'' - u' + 2u = 0, \quad u(0) = 2, \quad u'(0) = 0. \]

- (a) Find the solution \( u(t) \) of this problem.
- (b) find the first time at which \( |u(t)| = 10 \).

**Solution.**

(a) The characteristic equation \( 3r^2 - r + 2 = 0 \) has roots

\[ \frac{1}{6} + \frac{1}{6}i \sqrt{23}, \quad \frac{1}{6} - \frac{1}{6}i \sqrt{23}. \]

The general solution of the differential equation is

\[ u(t) = c_1 e^{(1/6)t} \cos\left(\frac{1}{6} \sqrt{23} t\right) + c_2 e^{(1/6)t} \sin\left(\frac{1}{6} \sqrt{23} t\right). \]

And

\[ u'(t) = \frac{1}{6} c_1 e^{(1/6)t} \cos\left(\frac{1}{6} \sqrt{23} t\right) - \frac{1}{6} c_1 e^{(1/6)t} \sin\left(\frac{1}{6} \sqrt{23} t\right) \sqrt{23}
\]
\[ + \frac{1}{6} c_2 e^{(1/6)t} \sin\left(\frac{1}{6} \sqrt{23} t\right) + \frac{1}{6} c_2 e^{(1/6)t} \cos\left(\frac{1}{6} \sqrt{23} t\right) \sqrt{23} \]

\( u(0) = 2 \) implies that

\[ c_1 = 2 \]

\( u(0) = 0 \) implies that

\[ \frac{c_1}{6} + \frac{c_2 \sqrt{23}}{6} = 0 \]

Thus,

\[ u(t) = 2 e^{(1/6)t} \cos\left(\frac{1}{6} \sqrt{23} t\right) - \frac{2}{23} \sqrt{23} e^{(1/6)t} \sin\left(\frac{1}{6} \sqrt{23} t\right) \]

(b) From the graph of \( u(t) \), we see that the first time \( |u(t)| = 10 \) happens when

\( u(t) = -10, \quad t = 10.75977 \)