Teaching to Inspire Mathematical Thinking

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A Story with a Lesson

- Eleven prospective elementary teachers
- Four basic mathematics items they couldn't solve to start

  1) LCM(60, 105) 2) GCD (60, 105)
  3) 0.4/.05 4) Multiplication of fractions story.
- One web-based teacher resource

Number of Correct Responses N = 11

Interview Tasks	Initial Inventory	Interview prior site	Interview after site	Final Inventory
# 1 -- GCD (60 , 105)	0	2	3	7
# 2 -- LCM (60, 105)	0	1	1	4
# 3 -- 0.4 /0.05	0	0	5	2
# 4 -- Fraction Mult.	0	3	4	7

What Happened at the Site?

- Almost no consideration of definitions.
- Little attention to explanations clarifying why procedures made sense.
- Little attention to notation or language.

Mathematical Sophistication

Values and ways of knowing are aligned with the mathematical community based on nine interwoven traits involving patterns, structures, conjectures, definitions, examples and models, relationships, arguments, language, and notation.

Sources of Conviction (from philosophy)
Mathematical Habits of Mind (Lim & Selden, 2009)
Executive Skills (Shoenfeld 1992)

An Example

Joe saw no tigers at the zoo.
Which of the following statements must be true?

a) All the tigers Joe saw were male.
b) All the tigers Joe saw were female.
c) Both of the above are statements are true.
d) None of the above statements is true.

Mathematical Sophistication Category 1: Patterns

"Seeing and revealing hidden patterns is what mathematicians do best."

L. Steen

"The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas like the colours or the words, must fit together in a harmonious way."

G. H. Hardy

Mathematical Sophistication Category 2: Structure

"Mathematics is the art of giving the same name to different things."

Poincaré

Mathematical Sophistication Category 3: Conjectures

"When you try to prove a theorem, you don't just list the hypothesis, and then start to reason. What you do is trial and error, experimentation, guesswork." Paul Halmos

"We [he and Halmos] share a philosophy about linear algebra: we think basis-free, we write basis-free , but when the chips are down we close the office door and compute with matrices like fury." Irving Kaplansky

Mathematical Sophistication Category 4: Models and Examples

"A good stock of examples, as large as possible, is indispensable for a thorough understanding of any concept, and when I want to learn something new, I make it my first job to build one."

Paul Halmos

Mathematical Sophistication Category 5: Definitions

"The mathematician is not concerned with the current meaning of his technical term…. The mathematical definition *creates* the mathematical meaning."

Polya

Mathematical Sophistication Category 6: Relationship

"Mathematicians do not study objects, but relations among objects; they are indifferent to the replacement of objects by others as long as relations do not change. Matter is not important, only form interests them."

Poincaré (as found in Gallian)

Mathematical Sophistication Category 7: Deduction

"Proof is the idol before whom the pure mathematician tortures himself."

Eddington

Mathematical Sophistication Category 8: Language

"Such is the advantage of a well-constructed language that its simplified notation often becomes the source of profound theories."

Laplace

Mathematical Sophistication Category 9: Notation

"In symbols one observes an advantage in discovery which is greatest when they express the exact nature of a thing briefly and, as it were, picture it; then indeed the labor of thought is wonderfully diminished."

Leibniz

3 + 7 = ____ + 8 = ____.
What numbers go in the blanks?

a) 10 and 18 respectively.
b) 2 and 10 respectively
c) Both of the above options work.
d) None of the above options works.

Why Teach Mathematics?

- To develop number and spatial sense.
- To convey specific definitions, procedures, theorems, and proofs.
- To teach problem solving and reasoning.
- To acculturate students – to make them mathematically sophisticated.

Thoughts on Acculturation from the World of Mathematics Education

There are educators that view all learning as a process of acculturation, and some of their ideas and research may prove useful.

How do Humans Learn?

Constructivist Learning Theory:

Individuals make things up based on our perceptions of the situation and expected modes of behavior (the culture), and based on prior knowledge and experiences. No one can do this for us. Every idea must be *conceived*.

"We don't see things as they are. We see things as *we* are."

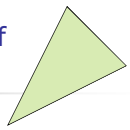
Anais Nin

Knowledge is Constructed by a Culture Based on our Purposes.

"Give up the requirement that knowledge represent an independent world, and admit instead that knowledge represents something that is far more important to us, namely what we can do in our experiential world, the successful ways of dealing with the objects we call physical and the successful ways of thinking with abstract concepts."

Ernst von Glasersfeld

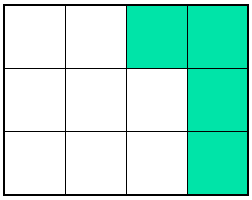
Humans Live in a World of Mental Objects



"It is impossible to distinguish and thus contrast the interpretation of the thing from the thing itself … because the interpretation of the thing *is* the thing."

Mehan & Wood

Pictures and Symbols do not Carry Meaning. Meaning Must be Negotiated.



Schipper (1982)

Context Matters

"Students arrive at what they know about mathematics mainly through participating in the social practice of the classroom… Students adapt their talk to the practice of their specific classroom."

H. Bauersfeld

Most Learning is Covert

"We have come to see the constructing of mathematical meaning as kind of subjective activity in which only a few parts are under actual conscious control. All the rest seems to be available instantaneously, "at hand," for the person and functions as orientation for action. This major part is subconsciously processed, and includes the use of language, symbols, and other means; the "framing of the situation"; how and when to do it; and so on."

H. Bauersfeld

What Does All This Suggest for Teaching to Inspire Mathematical Thinking?

"The only way to learn mathematics is to do mathematics."

Paul Halmos

1) Instructors must model the mathematical culture.
2) We need to make our values explicit.
3) We should negotiate meanings of problems and representations with students.
4) We should make use of "paradigm cases" to teach important lessons (like "=" example).

What might this look like in practice?

Jen's Weird Example of Doing Mathematics with my Class

These are elementary education majors who are mathematics minors.
Some of them know me.
It is the first day of class.
Someone is taking pictures.
My purpose is to acculturate them and to inspire mathematical thinking.

What is Mathematical Activity?

The Coin Problem

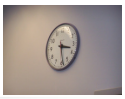
Suppose you have 12 identical coins. However one of them is counterfeit and weighs either more or less (you don't know which) than the others. You have a balance scale. How many weighings are needed to guarantee that you will identify the counterfeit coin? Prove it.

What Makes this a Good Problem to Inspire Mathematical Thinking?

- Not much mathematical machinery is needed to state the problem.
- Everyone can work on it, but it is unlikely that anyone will solve it in just one class period.
- Interesting mathematics arises.
- It can be extended or generalized.
- Ideally, you have not solved it yourself.

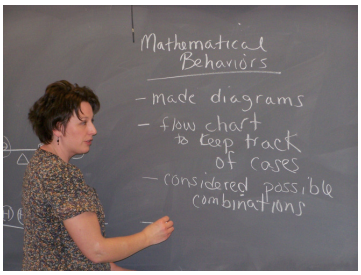
Giving support and time to make sense of the problem The room is silent for 10 minutes.

Then people begin to share ideas with their groups.



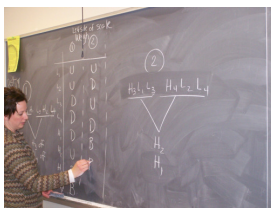
A first attempt at public solution proves inadequate. I am not the judge.

Students generated a list of things we did that were mathematical.



Don't be afraid to leave a problem unsolved – 47 hours later we had three solutions.

I encourage students to talk – even during a presentation - as long as the talk is mathematical.



What Happened?

I tried to model the mathematical culture.

We all engaged in mathematical thinking for at least 30 minutes.

We made features of that thinking explicit (and not just covert).

Most of us worked on the problem at home. Several of us confessed to doing it obsessively (in the shower, while driving, made our friends work on it).

We solved it (and lots of extension cases).

We practiced speaking and writing and listening to mathematics.

What's the Point of the Talk?

Only mathematicians can model the mathematical culture for students. It is up to us to acculturate them.

"Teachers also have to be exemplary, living models of the culture wanted, with transparent modi of thinking, reflecting, and self-controlling."
H. Bauersfeld

We want Mathematical Sophistication to be a residue that remains when the semester ends.

A Prize for Listening! Thank you.

The yellow sheets contain a compilation of
"Ten Great Problems"
to inspire mathematical thinking.

Please take one.

These slides are available at the UWO Mathematics Department webpage.

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