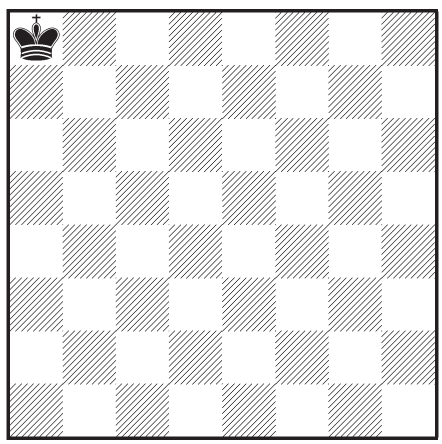
Exponential Growth: Doubling

A doubling application was posted on 11/26/2021 at `https://www.bbc.com` :

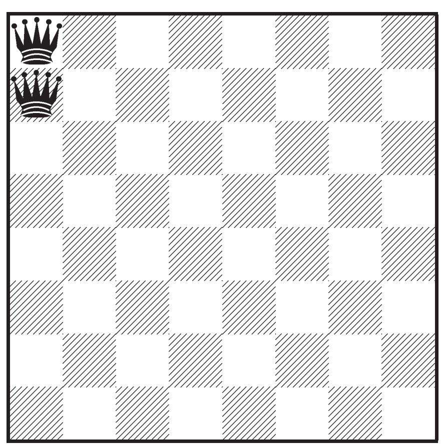
> **R value really quite high where variant was found - health chief**
>
> The new variant identified in South Africa is the most worrying we've seen yet, says the chief medical adviser of the UK Health Security Agency, Dr. Susan Hopkins. The epidemiologist says the R value in Gauteng, where the new variant was found, is now at 2 - which is "really quite high".
> ...the R value is the number of people that one infected person will pass on a virus to, on average. So in Gauteng, each infected person is on average passing it on to two more people.
> "We haven't seen levels of transmission like that since right back at the beginning of the pandemic because of all the mitigation and steps we've taken," she tells BBC Radio 4's Today programme.

For literature on *R value*, see the References on the last page.

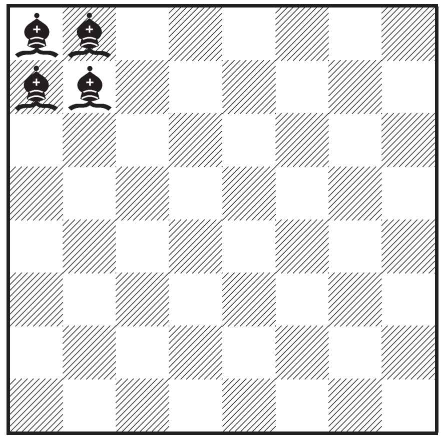
$2^{0} = 1$: Suppose we start on day 0 with one infected person:



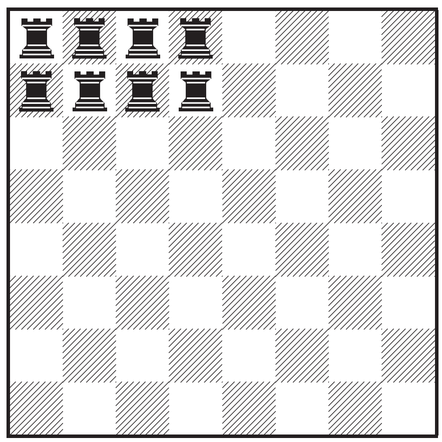
$2^{1} = 2$: Then on day 1 that one infects two more people, creating $2^1 = 2$ *new* cases:



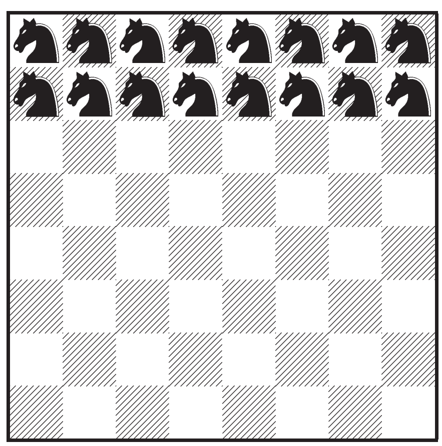
$2^{2} = 4$: Then on day 2 each of those two infects two more people, for $2^2 = 4$ *new* cases:



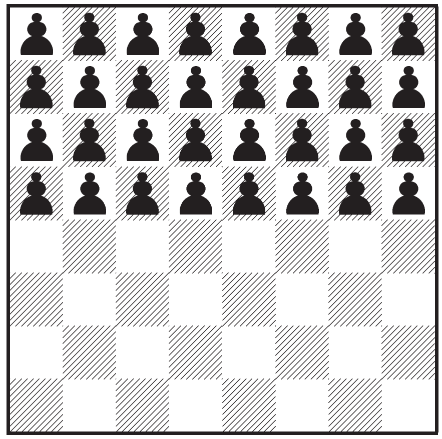
$2^{3} = 8$: Then on day 3 each of those four infects two more people, for $2^3 = 8$ *new* cases:



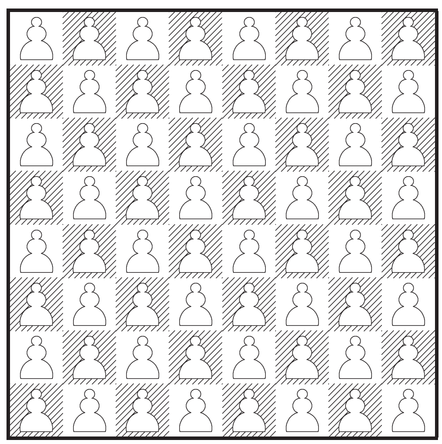
$2^{4} = 16$: Then on day 4 each of those eight infects two more people, for $2^4 = 16$ *new* cases:



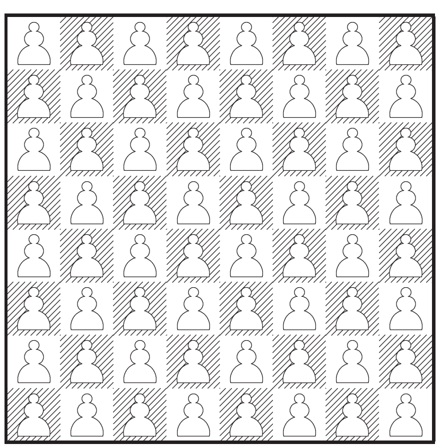
$2^{5} = 32$: Then on day 5 each of those 16 infects two more people, for $2^5 = 32$ *new* cases:

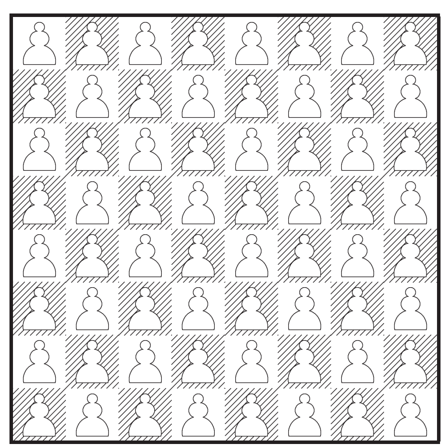


$2^{6} = 64$: Then on day 6 each of those 32 infects two more people, for $2^6 = 64$ *new* cases:



$2^{7} = 128$: Then on day 7 each of those 64 infects two more people, for $2^7 = 128$ *new* cases:





Exercise: Suppose the pattern continues this way, so that day n brings 2^n *new* cases. Then on which day will we have 1000 new cases? one million new cases?

For solutions: to go from $2^7 = 128$ to 1000 *new* cases, we may simply double a few more times. Or we may apply $\log_2$. See next page.

Exercise: Suppose the pattern continues this way, so that day n brings 2^n *new* cases. Then on which day will we have 1000 new cases? one million new cases?

Solutions:

1000 new cases:

$$\log_2(1000) = 3\log_2(10) = \frac{3\ln 10}{\ln 2} \cong 9.97$$

So day $\boxed{10}$ will bring over 1000 new cases.

In fact, day 10 will bring $2^{10} = 1024$ new cases.

one million $= 1,000,000 = 10^6$ new cases:

$$\log_2(10^6) = 6\log_2(10) = \frac{6\ln 10}{\ln 2} \cong 19.93$$

So doubling the number of new cases from one day to the next, on day $\boxed{20}$ we have over one million new cases.

In fact, day 20 will bring $2^{20} = 1,048,576$ new cases.

Continuing in this way, we see that at the end of 30 days (roughly one month), we will have $2^{30} = 1,073,741,824$ new cases.

Historical context:

> Once upon a time, computer professionals noticed that 2^{10} was very nearly equal to 1000 and started using the SI prefix "kilo" to mean 1024. That worked well enough for a decade or two because everybody who talked kilobytes knew that the term implied 1024 bytes. But, almost overnight a much more numerous "everybody" bought computers, and the trade computer professionals needed to talk to physicists and engineers and even to ordinary people, most of whom know that a kilometer is 1000 meters and a kilogram is 1000 grams.

– `https://physics.nist.gov/cuu/Units/binary.html`

This National Institute of Standards and Technology (NIST) site also clarifies what a megabyte is, and gives prefixes for many powers of 2.

The following items include clickable links to the articles. Our Math 104 canvas Literature module also has pdfs of these articles.

References

[1] Sebastian Contreras, Jonas Dehning, Matthias Loidolt, Johannes Zierenberg, F. Paul Spitzner, Jorge H. Urrea-Quintero, Sebastian B. Mohr, Michael Wilczek, Michael Wibral, Viola Priesemann, The challenges of containing SARS-CoV-2 via test-trace-and-isolate, *Nature Communications* (2021) 12:378. `https://doi.org/10.1038/s41467-020-20699-8`

[2] Paul L. Delamater, Erica J. Street, Timothy F. Leslie, Y. Tony Yang, Kathryn H. Jacobsen, Complexity of the Basic Reproduction Number (R_0), *Emerging Infectious Diseases* `www.cdc.gov/eid` (January 2019) Vol. 25, No. 1. `https://doi.org/10.3201/eid2501.171901`

[3] Luca Ferretti, Chris Wymant, Michelle Kendall, Lele Zhao, Anel Nurtay, Lucie Abeler-Dörner, Michael Parker, David Bonsall, Christophe Fraser, Quantifying SARS-CoV-2 transmission suggests epidemic control with digital contact tracing, *Science* (2020) 368, 6491. `https://www.science.org/doi/10.1126/science.abb6936`