Automatically Deriving Abstraction Heuristics

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About This Talk

Abstraction heuristics

Heuristic estimate is goal distance in abstracted state space S' obtained as homomorphism of original state space S.

Canonical example: pattern databases

Abstraction heuristics in the search community

A lot of thought has gone into developing (and analyzing) effective abstraction heuristics for particular search problems ($n^2 - 1$-puzzle, Rubik's Cube, Top Spin, ...).

This talk is about applying abstraction heuristics to problems where the search space is unknown to the algorithm designer.

Outline

Transition Systems

Definition (transition system)

A transition system is a 5-tuple $\langle S, L, A, s_0, S_\star \rangle$:

- S: finite set of states
- L: finite set of transition labels
- $A \subseteq S \times L \times S$: labelled transitions
- $s_0 \in S$: initial state
- $S_\star \subseteq S$: goal states

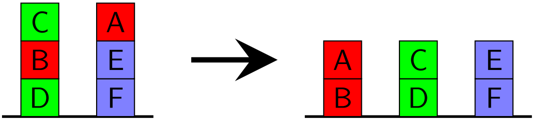
Objective: Find a shortest path from s_0 to some $s_\star \in S_\star$.

Factored Transition Systems

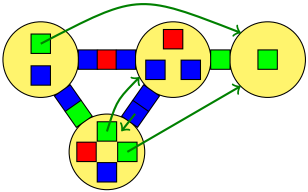
We assume a factored representation of transition systems:

- states: assignments to set $\mathcal{V}$ of state variables
- transitions and labels: given by set of operators defined in terms of a condition and effect on subsets of $\mathcal{V}$
- goal states: given by assignment to $\mathcal{V}' \subseteq \mathcal{V}$

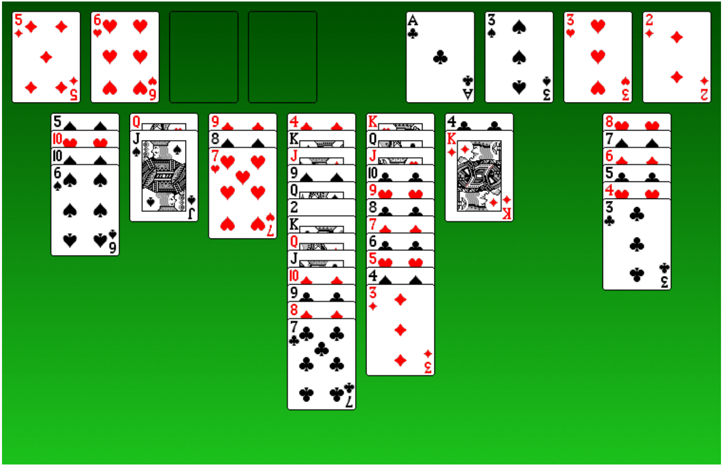
Example: Blocksworld



Example: Pipesworld



Example: FreeCell



Abstractions

Definition (abstraction, homomorphism)

Abstraction of transition system $\mathcal{T}$: pair $\langle \mathcal{T}', \alpha \rangle$ where

- $\mathcal{T}'$ is a transition system with the same labels
- α maps states of $\mathcal{T}$ to states of $\mathcal{T}'$ such that
  - initial state maps to initial state
  - goal states map to goal states
  - transitions $\langle s, l, s' \rangle$ map to transitions $\langle \alpha(s), l, \alpha(s') \rangle$

Abstraction (and α) is a homomorphism if $\mathcal{T}'$ only contains necessary goal states and transitions.

Abstraction heuristic: $h(s) = d_\star(\alpha(s))$ admissible, consistent

Generating Abstractions

Conflicting goals in generating abstractions:

- obtain informative heuristic
- keep representation small

Abstractions have small representations if they have

- few abstract states
- succinct encoding for α

Pattern Databases

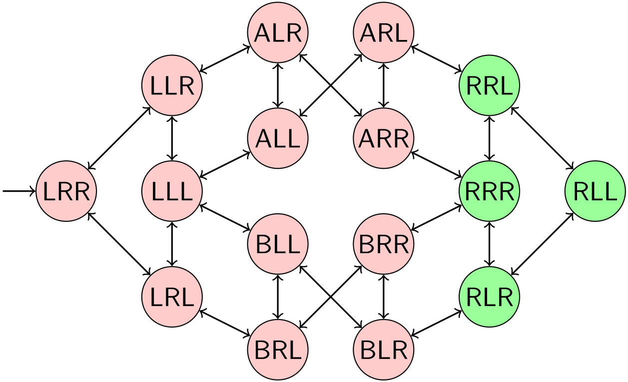
One idea to get succinct encodings: projections
$\rightsquigarrow$ map states to abstract states with perfect hash function

Definition (projection)

Projection $\pi_{\mathcal{V}'}$ to variables $\mathcal{V}' \subseteq \mathcal{V}$:
homomorphism α where $\alpha(s) = \alpha(s')$ iff s and s' agree on $\mathcal{V}'$

Abstraction heuristics for projections are called
pattern database (PDB) heuristics.

Example: Transition System

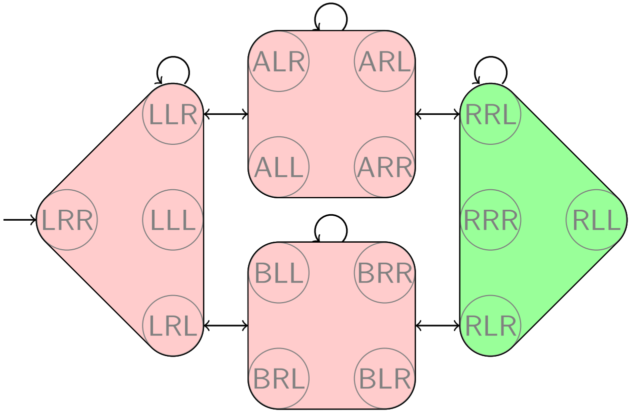


Logistics problem with one package, two trucks, two locations:

- state variable package: $\{L, R, A, B\}$
- state variable truck A: $\{L, R\}$
- state variable truck B: $\{L, R\}$

Example: Projection

Project to {package}:



Automatically Derived Abstraction Heuristics

Our research problem

Automatically derive an effective abstraction heuristic
for a given transition system in factored representation.

Some important papers:

- Edelkamp (ECP-01): Planning with PDBs
- Edelkamp (AIPS-02): Symbolic PDBs
- Haslum et al. (AAAI-05): Constrained PDBs
- Haslum et al. (AAAI-07): Pattern selection
- Helmert et al. (ICAPS-07): Explicit-state abstractions
- Katz & Domshlak (ICAPS-08): Optimal cost partitioning
- Katz & Domshlak (ICAPS-08): Structural patterns

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Outline

Reference

This part based on:

Patrik Haslum, Adi Botea, Malte Helmert, Blai Bonet, Sven Koenig.
Domain-Independent Construction of Pattern Database Heuristics for Cost-Optimal Planning.
Proc. AAAI 2007, pp. 1007–1012, 2007.

PDB Abstractions for Factored Transition Systems

Objective

Automatically derive an effective pattern database heuristic for a given transition system in factored representation.

Guiding questions:

1. What is a pattern for a factored transition system?
2. How can we identify and exploit disjunctive patterns?
3. Which patterns do we choose?

Patterns for Factored Transition Systems

1. What is a pattern for a factored transition system?

Most natural definition:

- identify patterns with sets of state variables to project to

Definition (abstracted transition system)

Let $\mathcal{T}$ be a factored transition system and $\mathcal{V}$ its variable set. Let $P \subseteq \mathcal{V}$ be a pattern.

The abstracted transition system $\mathcal{T}(P)$ is obtained from $\mathcal{T}$ by

- restricting the initial state to P
- restricting operator conditions and effects to P
- removing goal conditions on variables not in P

Pattern Heuristics

Definition (pattern heuristic)

Let $\mathcal{T}$ be a factored transition system and $\mathcal{V}$ its variable set.
Let $P \subseteq \mathcal{V}$ be a pattern.

The pattern heuristic h^P assigns to each state s of $\mathcal{T}$ the length of an optimal solution for $\mathcal{T}(P)$, starting from the state obtained by restricting s to P.

For all choices of P, heuristic h^P is admissible and consistent.

What can we do if we have multiple patterns $P_1, \ldots, P_k$?

Criterion for Disjunctive Patterns

2. How can we identify and exploit disjunctive patterns?

Theorem (disjunctive patterns)

Let $\mathcal{C}$ be a pattern collection, i.e. a set of patterns of task $\mathcal{T}$. We say that an operator affects a pattern P if it can assign a new value to some variable $v \in P$.

If no operator in $\mathcal{T}$ affects more than one pattern in $\mathcal{C}$, then $\sum_{P \in \mathcal{C}} h^P$ is admissible and consistent.

Finding Disjunctive Patterns

Finding sets of disjunctive patterns in a pattern collection $\mathcal{C}$:

- build compatibility graph for $\mathcal{C}$
  - vertices correspond to patterns $P \in \mathcal{C}$
  - edge between two vertices iff no operator affects both
- compute all maximal cliques of the graph
  using the algorithm of Tomita, Tanaka & Takahashi

The Canonical Heuristic Function

Definition (canonical heuristic function)

Let $\mathcal{T}$ be a factored transition system and $\mathcal{V}$ its variable set.
Let $\mathcal{C}$ be a pattern collection.

The canonical heuristic $h^{\mathcal{C}}$ for pattern collection $\mathcal{C}$ is defined as

$$h^{\mathcal{C}}(s) = \max_{\mathcal{D} \in \text{cliques}(\mathcal{C})} \sum_{P \in \mathcal{D}} h^{P}(s),$$

where cliques$(\mathcal{C})$ is the set of all maximal cliques in the compatibility graph for $\mathcal{C}$.

For all choices of $\mathcal{C}$, heuristic $h^{\mathcal{C}}$ is admissible and consistent.

It is the best possible admissible heuristic that can be derived from the information in the pattern databases in $\mathcal{C}$.

The full story includes "dominance pruning" to optimize speed.

Building the Pattern Collection

3. Which patterns do we choose?

$\leadsto$ perform local search in the space of pattern collections

- search method:
  hill-climbing (steepest ascent)
- initial collection:
  $\{\{v\} \mid v \in \mathcal{V}, v$ has a goal condition$\}$
- search neighbourhood:
  for each pattern $P \in \mathcal{C}$ and each variable $v \in \mathcal{V}$,
  the collection $\mathcal{C} \cup \{P \cup \{v\}\}$ is a neighbour
  unless the pattern database for $P \cup \{v\}$ would exceed a pre-specified memory limit
- evaluation function:
  estimate heuristic quality of collection (next slide)

Estimating Heuristic Quality

- for pattern collection $\mathcal{C}$, want to estimate the quality of $h^{\mathcal{C}}$
- only need to estimate degree of improvement:
  - For neighbours $\mathcal{C}_1$ and $\mathcal{C}_2$ of $\mathcal{C}$, which of $h^{\mathcal{C}_1}$ and $h^{\mathcal{C}_2}$ leads to the larger improvement over $h^{\mathcal{C}}$?
- using an analytical model for heuristic search performance by Korf, Reid & Edelkamp (and some simplifying assumptions), this is reduced to:
  - Which of $h^{\mathcal{C}_1}$ and $h^{\mathcal{C}_2}$ has a higher probability of giving a better estimate than $h^{\mathcal{C}}$ for randomly drawn states?

    using a particular non-uniform random distribution that I do not want to discuss in detail...

$\rightsquigarrow$ problem reduces to computing $h^{\mathcal{C}_i}(s)$ for some states s

Estimating Heuristic Quality (ctd.)

$\rightsquigarrow$ problem reduces to computing $h^{\mathcal{C}_i}(s)$ for some states s

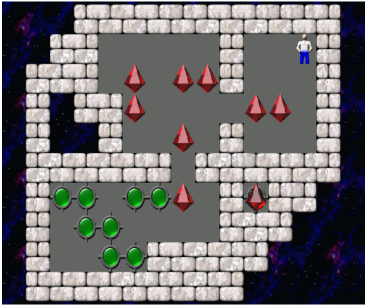
- Idea: compute $h^{\mathcal{C}_i}(s)$ without computing the pattern database for each new pattern (too expensive)

$\rightsquigarrow$ perform A* searches in the state space of $h^{P \cup \{v\}}$ using h^P as a heuristic

Empirical Evaluation

We tested the approach on

- 24 instances of the 15-puzzle and
- 40 Sokoban instances.



Empirical Evaluation: Results

15-puzzle:

- we can solve all 24 instances optimally
- compared to the only other known general algorithm which manages this (Haslum, Bonet & Geffner 2005):
  - their technique: 2,559,508 node expansions
  - our technique: 549,147 node expansions

Sokoban:

- we can solve 23 out of 40 instances optimally
- we are not aware of any other general algorithm which can solve any of these optimally

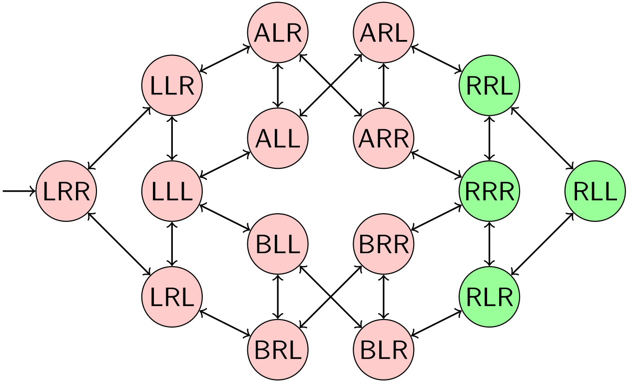
Outline

References

This part based on:

Malte Helmert, Patrik Haslum, Jörg Hoffmann.
Flexible Abstraction Heuristics for Optimal Sequential Planning.
Proc. ICAPS 2007, pp. 176–183, 2007.

Example: Transition System



Logistics problem with one package, two trucks, two locations:

- state variable package: $\{L, R, A, B\}$
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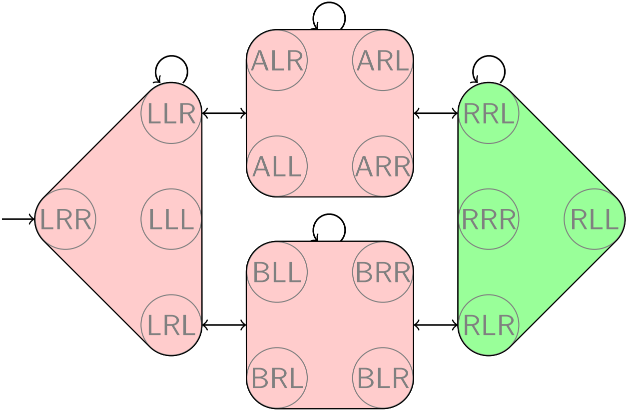
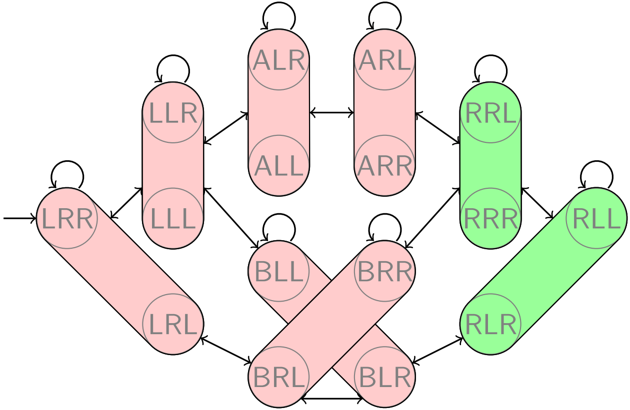
Example: Projection

Project to {package}:



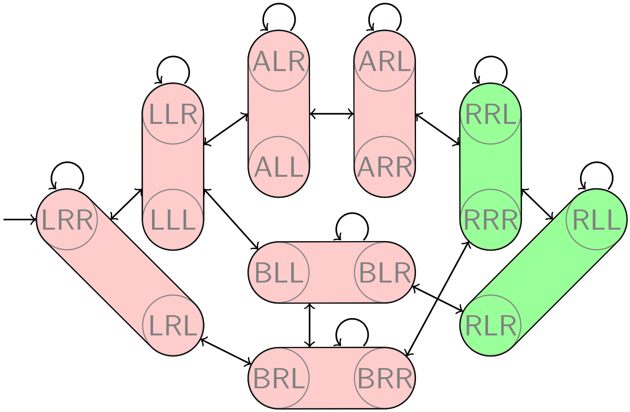
Example: Projection (2)

Project to $\{\text{package}, \text{truck A}\}$:



Example: Projection (2)

Project to {package, truck A}:



Limitations of Projections

How accurate is the PDB heuristic?

- consider generalization of the example:
  N trucks, M locations (still one package)
- consider any pattern that is proper subset of $\mathcal{V}$
- $h(s_0) \leq 2 \rightsquigarrow$ no better than atomic projection to package

(maximizing over patterns or disjunctive patterns do not help either)

Explicit-State Abstraction Heuristics: Main Idea

Main idea

(due to Dräger, Finkbeiner & Podelski, 2006):

Instead of perfectly reflecting a few state variables, reflect all state variables, but in a potentially lossy way.

Explicit-State Abstraction Heuristics: Key Insights

Key insights:

1. Information of two abstractions $\mathcal{A}$ and $\mathcal{A}'$ of the same transition system can be composed by a simple graph-theoretic operation (synchronized product $\mathcal{A} \otimes \mathcal{A}'$).
2. The complete state space can be recovered using only atomic projections:

$$\bigotimes_{v \in \mathcal{V}} \pi_v \text{ is isomorphic to } \pi_{\mathcal{V}}.$$

$\rightsquigarrow$ build fine-grained abstractions from coarse ones

3. When intermediate results become too big, we can shrink them by aggregating some abstract states.

Computing Explicit-State Abstractions

Generic abstraction computation algorithm

abs := all atomic projections π_v $(v \in \mathcal{V})$.
while abs contains more than one abstraction:
    select $\mathcal{A}_1$, $\mathcal{A}_2$ **from** abs
    shrink $\mathcal{A}_1$ and/or $\mathcal{A}_2$ until $\mathit{size}(\mathcal{A}_1) \cdot \mathit{size}(\mathcal{A}_2) \leq N$
    abs := abs $\setminus \{\mathcal{A}_1, \mathcal{A}_2\} \cup \{\mathcal{A}_1 \otimes \mathcal{A}_2\}$
return the remaining abstraction

N: parameter bounding number of abstract states

Questions for practical implementation:

- Which abstractions to select? $\rightsquigarrow$ composition strategy
- How to shrink an abstraction? $\rightsquigarrow$ shrinking strategy
- How to choose N?

Experimental Evaluation

Comparison to state of the art

Is this competitive with the state of the art?

- Compare scaling behaviour to other heuristics: blind, $h^{\max}$, PDB

Domain	**abs**	blind	$h^{\max}$	PDB
Pipes-NoTankage	**19**	14	15	15
Pipes-Tankage	**13**	10	10	7
Satellite	**6**	4	5	6
Logistics	**18**	6	6	16
PSR	**5**	3	4	4
TPP	**7**	5	6	6
total	**68**	42	46	54

Theoretical Properties

As powerful as PDBs

PDB heuristics are a special case of our abstraction heuristics, and arise naturally as a side product.

Get additivity for free

If P and P' are disjunctive patterns, then
for all h-preserving abstractions $\mathcal{A}$ of π_P and $\mathcal{A}'$ of $\pi_{P'}$,
the abstraction heuristic for $\mathcal{A} \otimes \mathcal{A}'$ dominates $h^P + h^{P'}$.

Greater representational power

In some planning domains where PDBs have unbounded error (GRIPPER, SCHEDULE, two PROMELA variants), we can obtain perfect heuristics in polynomial time with suitable composition/shrinking strategies.

Outline

Conclusion

Take-home message

Abstraction heuristics are equally useful
in settings where they must be automatically derived
as in settings where they can be hand-tailored.

Moreover, there are plenty of research opportunities
in transferring results from one area to the other.

Come to ICAPS 2008!

Tutorial:

- Abstraction Heuristics for Planning: PDBs and Beyond
  (Patrik Haslum and myself)

Papers:

- The Compression Power of Symbolic Pattern Databases
  (Marcel Ball and Robert C. Holte)
- Optimal Additive Composition of Abstraction-based Admissible Heuristics
  (Michael Katz and Carmel Domshlak)
- Structural Patterns Heuristics via Fork Decomposition
  (Michael Katz and Carmel Domshlak)