MATH 206 Spring 2015

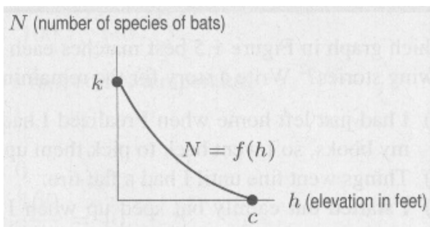
Homework Assignments

Homework 1, Due February 13

1) In the Andes mountains in Peru, the number, N, of species of bats in a region is a function of the elevation, h, in feet above sea level, so $N = f(h)$.

   a) Write a sentence interpreting the statement $f(500) = 100$ in terms of bat species and elevation.

   b) Write sentences explaining the meaning of the vertical intercept, k, and of the horizontal intercept, c, in the graph below.



2) You drive at a constant speed from Chicago to Detroit, a distance of 275 miles. About 120 miles from Chicago you pass through Kalamazoo, Michigan.

   a) Choose a specific speed (such as 60 mph) and sketch a graph of your distance from Kalamazoo as a function of time since your trip began.

   b) How does the *shape* of your graph change for a speed that is 10% faster than the one you chose for part a)? Explain.

3) A company rents cars at a daily rate of \$40 and also charges 15¢ per mile traveled. Its competitor has a daily rate of \$50 but only charges 10¢ per mile.

   a) Give formulas for each company's fee as a function of the distance traveled.

   b) Graph both functions on suitable axes. (Your intent is to use the graph to see which company is a better choice for various day-trip distances.)

   c) Use your graph in part b) to describe how you decide which company to choose.

4) In a California town, the monthly charge for waste collection is \$8 for 32 gallons of waste and \$12.32 for 68 gallons of waste.

   a) Find a linear function for the cost, C, of waste collection as a function of the number of gallons of waste, w.

   b) Using appropriate units, write a sentence interpreting the slope your line.

   c) Using appropriate units, write a sentence interpreting the intercept your line.

5) Do you expect the average rate of change (in units per year) of each of the following to be positive, negative, or zero? Explain your reasoning.

   a) Number of acres of rain forest in the world.

   b) Population of the world.

   c) Number of polio cases each year in the US, since 1950.

   d) Height of a sand dune that is being eroded.

   e) Cost of living in the US.

Homework 2, Due February 20

1) When the price, p, charged for a boat tour was \$25, the average number of passengers per week, N, was 500. When the price was reduced to \$20, the average number of passengers per week increased to 650. Find a formula for the demand function, as a function of price, assuming it is linear.

2) Determine whether each of the following functions could be represented *exactly* with a linear function, an exponential function, or neither function. Explain how you know. Find a formula for the function if you determined it was either linear or exponential.

a)

x	0	1	2	3
$f(x)$	10.5	12.7	18.9	36.7

b)

t	–1	0	1	2
$s(t)$	50.2	20.12	18.072	10.8432

c)

u	0	2	4	6
$g(u)$	27	24	21	18

3) During a recession a firm's revenue declines continuously so that the revenue, R, measured in millions of dollars, in t years' time is estimated by $R = 5e^{-0.15t}$.

   a) Indicate the current revenue and calculate the estimated revenue in two years' time.

   b) After how many years will the estimated revenue have declined to \$2.7 million?

4) You win \$38,000 in the state lottery, to be paid in two \$19,000 installments: one now and one in a year. A friend offers you \$36,000 now in return for your lottery ticket. Instead of accepting your friend's offer, you could take out a one-year loan at an interest rate of 8.25% per year, compounded annually. The loan will be paid back by a single payment of \$19,000 (your second lottery check) at the end of the year. Which is a better deal for you, your friend's offer of \$36,000, or your plan of taking the loan plus your first check of \$19,000? [Hint: Figure out what the bank will let you borrow.]

5) Complete the following table:

x	–3	–2	–1	0	1	2	3
$f(x)$	0	1	2	3	2	1	0
$g(x)$	3	2	2	0	–2	–2	–3
$f(g(x))$							
$g(f(x))$							

Homework 3, Due March 6

1) Allometry is the study of the relative size of different parts of a body as a consequence of growth. In this problem, you will check the accuracy of an allometric hypothesis: the weight of a fish is proportional to the cube of its length. The data below show the weight, w, in gm, of plaice (a type of fish) to its length, l, in cm. Does the data support the hypothesis that $w = kl^3$ (approximately)? If so, estimate the constant of proportionality, k.

l	33.5	35.5	37.5	39.5	41.5	43.5
w	332	391	455	538	623	724

2) The data below gives $P = f(t)$, the number of households, in millions, in the US with cable television t years since 1998.

t	0	2	4	6	8	10
P	64.65	66.25	66.73	65.73	65.14	64.87

a) Does $f'(4)$ appear to be positive or negative or zero? Write a sentence interpreting the importance of this result.

b) Estimate $f'(2)$ and $f'(10)$ each time writing a sentence interpreting the value.

3) Complete the following table:

x	0	5	10	15	20
$f(x)$	100	70	55	46	40
$f'(x)$					

4) The average weight, W, in pounds, of an adult is a function, $W = f(c)$, of the average number of calories per day, c, consumed.

a) Write a sentence interpreting the statements $f(1800) = 155$ and $f'(2000) = 0$ in terms of daily caloric consumption and weight.

b) What are the units of $f'(c) = dW/dc$?

5) The data below shows $N = f(t)$, the number of Facebook subscribers, in millions, worldwide at 3-month intervals.

Month, t	Mar. 2011	Jun. 2011	Sep. 2011	Dec. 2011	Mar. 2012
N	664.0	710.7	756.9	799.1	835.5

a) Calculate the average rate of change of N per month for the time intervals shown. How do these values relate to $f'(t)$?

b) What can you observe about the sign of d^2n/dt^2 for the 12-month period between March, 2011 and March, 2012? [Hint: Calculate a few values of the second derivative using your values from part a).]

Homework 4, Due March 17

1) A company's cost of producing q liters of a chemical is $C(q)$ dollars and this quantity can be sold for $R(q)$ dollars. Suppose $C(2{,}000) = 5{,}930$ and $R(2{,}000) = 7{,}780$.

a) What is the profit at a production level of 2000?

b) If $MC(2{,}000) = 2.1$ and $MR(2{,}000) = 2.5$, what is the approximate change in profit if q is increased from 2000 to 2001? Should the company increase or decrease production from $q = 2000$ and why?

c) If $MC(2{,}000) = 4.77$ and $MR(2{,}000) = 4.32$, should the company increase or decrease production from $q = 2{,}000$ and why?

2) The demand for a product is given by $p = f(q) = 50 - 0.03q^2$, where p is in dollars and q is in thousands of units.

a) Find the p- and q-intercepts for this function and write a sentence for each interpreting them in terms of demand for this product.

b) Find $f(20)$ and give units with your answer. Explain what it tells you about demand.

c) Find $f'(20)$ and give units with your answer. Explain what it tells you about demand.

3) In 2009, the population of Hungary was approximated by $P = 9.906(0.997)^t$, where P is in millions of people and t is in years since 2009. Assume the trend has continued and will continue in the future.

a) What does this model predict for the population of Hungary in the year 2020?

b) At what rate does this model predict the population of Hungary will be changing in the year 2020? Include units with your answer.

4) A firm estimates that the total revenue, R, received from the sale of q items is given by $R = \ln(1 + 1000q^2)$. Calculate the exact marginal revenue when $q = 10$ and then approximate the marginal revenue using the derivative.

5) Find the equation of the tangent line to the graph of $f(x) = \frac{2x-5}{x+1}$ at $x = 0$. Check your answer by graphing the function and your equation on the same axes and verifying visually that it is indeed the tangent line at $x = 0$. [Hint: You must find $f'(x)$ first to get the slope of the line.]

Homework 5, Due March 31

1) Suppose f has a continuous derivative whose values are given in the following table.

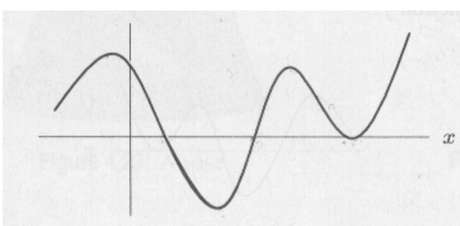
x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	5	2	1	–2	–5	–3	–1	2	3	1	–1

a) Estimate the x-coordinates of critical points of f for $0 \le x \le 10$.

b) For each critical point, indicate if it is a local maximum of f, a local minimum, or neither and explain your reasoning.

2) Find all critical points for the function $y = x^3 - ax^2$. Sketch several members of the family on the same axes. Discuss the effect of the parameter a on the graph.

3) Using the figure below, indicate approximately where the inflection points are if the graph shows:



a) The graph given is a graph of $f(x)$.

b) The graph given is a graph of the derivative $f'(x)$.

c) The graph given is a graph of the second derivative $f''(x)$.

4) The function $y = t(x)$ is positive everywhere, continuous everywhere, and has a global maximum at the point (3,3). Sketch a possible graph of $t(x)$ if $t'(x)$ and $t''(x)$ have the same sign for $x < 3$, but opposite signs for $x > 3$. [Hint: The global maximum might *not* have a derivative value of zero.]

5) If you have 100 feet of fencing and want to enclose a rectangular area up against an existing straight wall (longer than 100 feet), what is the largest area you can enclose? (Assume you use all of your fencing.) How does your answer change if you require the enclosure to be square instead of just rectangular? [Hint for both parts: First write the area as a function of one side. Then use derivatives.]

Homework 6, Due April 13

1) A demand function for a product is $p = 400 - 2q$, where q is the quantity sold for price $\$p$.

a) Find an expression for the total revenue, R, as a function of q. [Hint: Recall that in general $R = pq$.]

b) Differentiate R with respect to q to find the approximate marginal revenue, MR, as a function of q. Estimate the marginal revenue when $q = 10$.

c) Calculate the change in total revenue when production increases from $q = 10$ to $q = 11$ units. Confirm that the estimated MR from part b) is approximately equal to the change in revenue from a one-unit increase in production, the actual MR.

2) You are the manager of a firm that produces slippers that sell for \$20 a pair. You are producing 1,200 pairs of slippers each month, at an average cost of \$10 each. The marginal cost at a production level of 1,200 is \$12 per pair.

a) Are you making or losing money?

b) Will increasing production increase or decrease your average cost? Your profit?

c) Would you recommend that production be increased or decreased?

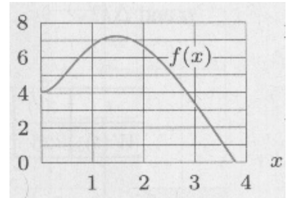
3) An old rowboat has sprung a leak. Water is flowing into the boat at a rate, $r(t)$, given in the table below.

t minutes	0	5	10	15
$r(t)$ liters / minute	12	20	24	16

a) Compute upper and lower estimates for the volume of water that has flowed into the boat during the 15 minutes.

b) Draw a graph that illustrates the *lower* estimate you calculated in part a).

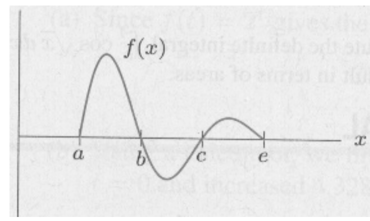
4) Use the graph below to estimate $\int_0^3 f(x)dx$. (If you count boxes, be sure to be clear about how you dealt with partial boxes.)



5) Using the graph below, rank the following five integrals in order from smallest value to largest. (Recall that –10 is smaller than –5.) Also indicate which integrals are negative and which are positive. Explain your reasoning.

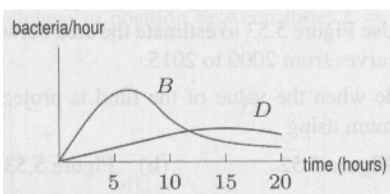
I. $\int_a^b f(x)dx$ II. $\int_a^c f(x)dx$ III. $\int_a^e f(x)dx$

IV. $\int_b^e f(x)dx$ V. $\int_b^c f(x)dx$



Homework 7, Due April 21

1) The birth rate, B, in births per hour, of a bacteria population is given in the figure below. The other curve, D, gives the death rate of the same population, in deaths per hour.

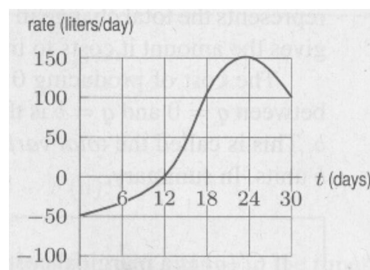


a) Explain what the shape of each of these graphs tells you about the population.

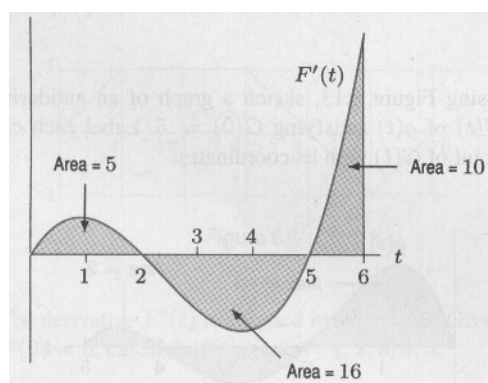
b) Use the graphs to find the time at which the net rate of increase of the population is at a maximum. Explain your reasoning.

c) At time $t = 0$ the population has size N. Sketch the graph of the total number born *by* time t. Also sketch the graph of the number alive *at* time t. Estimate the time at which the population is a maximum.

2) The figure below shows the rate of change of the quantity of water in a water tower, in liters per day, during the month of April. If the tower had 12,000 liters of water in it on April 1, estimate the quantity of water in the tower on April 30.



3) The graph below shows the derivative $F'(t)$. If $F(0) = 3$, find the values of $F(2)$, $F(5)$, and $F(6)$. Graph $F(t)$. [Note: Make sure you graph the critical points of $F(t)$ fairly; i.e., don't draw it flat if the derivative is not zero.]



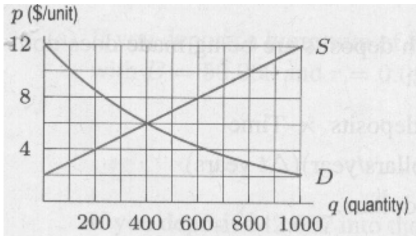
4) Find the indefinite integral: $\int \left(x^2 + \frac{1}{x}\right) dx$.

5) A firm's marginal cost function is $MC = 3q^2 + 4q + 6$. Find the total cost function if the fixed costs are 200.

Homework 8, Due April 29

1) Evaluate using the FTC: $\int_0^1 (6q^2 + 4)dq$.

2) The supply and demand curves for a product are given in the figure below.



a) Estimate the equilibrium price and quantity.

b) Estimate the consumer surplus and the producer surplus. [Warning: Using a triangle for demand will over-estimate the value.]

c) Estimate the consumer surplus and the producer surplus if the price is artificially set low at $p^- = 4$ dollars per unit. Compare your answers to part b) above. Do the new values make sense?

3) Your company needs $500,000 in two years' time for renovations and can earn 9% on investments.

a) What is the present value of the renovations?

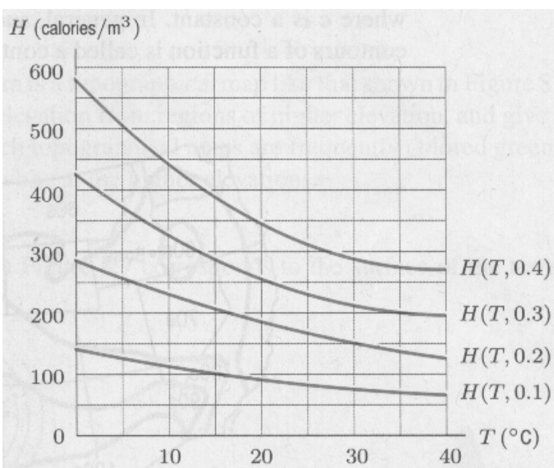
b) If your company deposits money continuously at a constant rate throughout the two-year period, at what monthly rate should the money be deposited so that you have the $500,000 when you need it?

4) Evaluate using substitution: $\int \frac{t}{1+3t^2} dt$.

5) Evaluate using substitution: $\int x\left(\sqrt{3x^2+4}\right) dx$.

Homework 9, Due May 11

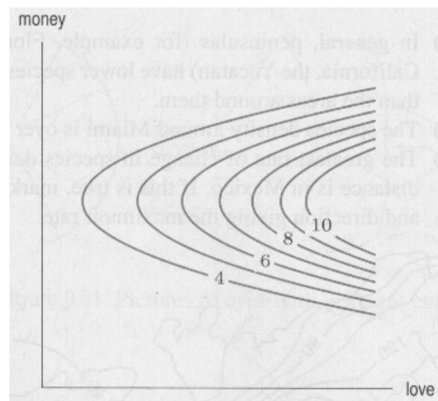
1) An airport can be cleared of fog by heating the air. The amount of heat required, $H(T, w)$ (in calories per cubic meter of fog), depends on the temperature of the air, T (in °C), and the wetness of the fog w (in grams per cubic meter of fog). The figure below shows several cross-sections of H against T with w fixed.



a) Estimate $H(20, 0.3)$ and explain what information it gives us.

b) Make a table of values for $H(T, w)$. Use $T = 0, 10, 20, 30$, and 40, and $w = 0.1, 0.2, 0.3$, and 0.4.

2) The contour diagram below shows your happiness as a function of love and money.

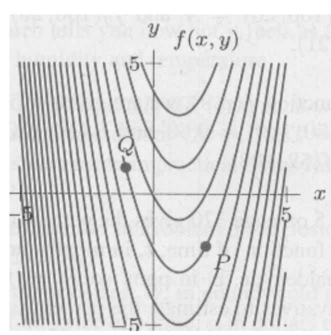


a) Describe in words your happiness as a function of money, with love at a fixed level.

b) Describe in words your happiness as a function of love, with money at a fixed level.

c) Graph two different cross-sections with love fixed and two different cross-sections with money fixed.

3) The figure below shows contours of $f(x, y)$ with values of f on the contours omitted. There are two points labeled on the figure, P and Q. Assume that $f_x(P) > 0$.



a) What is the sign of $f_y(P)$? Explain.

b) What is the sign of $f_y(Q)$? Explain.

c) What is the sign of $f_x(Q)$? Explain.

4) Calculate all four second-order partial derivatives and confirm that the mixed partials are equal, using $f(x, t) = t^3 - 4x^2 t$.

5) A missile has a guidance device which is sensitive to both temperature, t, in °C, and humidity, h, in percent. The range, R, in km over which the missile can be controlled is given by $R(t, h) = 27{,}800 - 5t^2 - 6ht - 3h^2 + 400t + 300h$. What are the optimal atmospheric conditions for controlling the missile, and what is the maximum range R over which it can be controlled?