67-717 Nonlinear Dynamics and Chaos
Summer 2007

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Office Hours: MTWR: 8:30-9:00; MTWR: 12:00-1:00 and by appointment.

Course Content:

This course is an introduction to the study of dynamical systems. Nonlinear differential equations and iterative maps arise in the mathematical description of numerous systems throughout science and engineering, for instance in physics, chemistry, biology, economics, and elsewhere. Such systems may display complicated and rich dynamical behavior, and we will develop some linear and nonlinear mathematical tools for their analysis, and consider models in such fields as population biology, ecology, and mechanical and electrical oscillations. Our emphasis throughout will be on the qualitative behavior of the models, in particular, on the prediction of qualitative change in the nature of the dynamics as a system parameter varies (bifurcation). The course will stress conceptual ideas, geometric intuition and concrete examples and applications. The mathematical treatment will be friendly and informal but still careful.

In this course we will proceed from simpler to more complicated (and more interesting!) systems. We begin with one-dimensional flows, their steady states, stability and bifurcations, and then observe the far more complicated dynamics, including chaos, that may occur in one-dimensional maps. Phase-plane analysis in two dimensions reveals the possibility of oscillations and limit cycles, and we study their bifurcations. As time permits, we will also investigate higher-dimensional dynamical systems, deterministic chaos and strange attractors. By the end of the course you will understand terms such as bifurcations, limit cycles, Lorenz equations, chaos, iterated maps, period doubling, fractals and strange attractors. Along the way we will consider interesting and important applications such as mechanical vibrations, lasers, biological rhythms, superconducting circuits, insect outbreaks, chemical oscillators, chaotic waterwheels, and even a technique for using chaos to send secret messages.

Text: S. Strogatz_(1994): *Nonlinear Dynamics and Chaos* by, 1st edition, Westview Press. ISBN: 0-7382-0453-6.

Topics to be covered:

One-Dimensional Flows:

1. One dimensional Flows: Fixed Points and Stability, Population Growth, Linear Stability Analysis, Existence and Uniqueness of Solutions, Potentials, Solving Equations on the Computer.
2. Bifurcations: Saddle-Node Bifurcation, Transcritical Bifurcation, Pitchfork Bifurcation, Overdamped Bead on a Rotating Hoop, Insect Outbreak.

3. Flows on the Circle: Uniform and Nonuniform Oscillators, Overdamped Pendulum, Fireflies, Superconducting Josephson Junctions.

Two-Dimensional Flows:

1. Linear and Nonlinear Systems : Classification, Phase Plane, Phase Portrait, Fixed Points and Linearization, Rabbit versus Sheep, Reversible Systems.
2. Limit Cycles: Poincare-Bendixson Theorem, Lienard Systems, Relaxation oscillators
3. Bifurcation: Saddle-Node, Transcritical and Pitchfork Bifurcations, Hopf Bifurcations, Oscillating Chemical Reactions, Poincare Maps.

Chaos:

1. Lorenz Equations: Chaotic Waterwheel, Chaos on a Strange Attractor, Lorenz Map.
2. One-Dimensional Maps: Fixed Points and Cobwebs, Analysis and Numerics of the Logistic Map, Periodic windows, Liapunov Exponents.
3. Fractals: Countable and Uncountable Sets, Cantor Set, Dimension of Self-Similar Fractals, Box Dimension.
4. Strange Attractors: The Simplest examples, Henon Map, Rossler Map, Chemical Chaos.

Recommended Reading:

Popularizations:

1. Gleick, J. (1987). Chaos, the Making of a New Science.London, Heinemann

2. Stewart, I. (1989). Does God Play Dice? Cambridge, Blackwell.

3. Devaney, R. L. (1990). Chaos, Fractals, and Dynamics: Computer Experiments in Mathematics. Menlo Park, Addison-Wesley

4. Lorenz, E., (1994) The Essence of Chaos, Univ. of Washington Press

5. Schroeder, M. (1991) Fractals, Chaos, Power: Minutes from an infinite paradise W. H. Freeman New York:

Intermediate Texts:

1. Devaney, R. L. (1986). An Introduction to Chaotic Dynamical Systems. Menlo Park, Benjamin/Cummings.

2. Kaplan, D. and L. Glass (1995). Understanding Nonlinear Dynamics, Springer-Verlag New York.

3. Jurgens, H., H.-O. Peitgen, et al. (1993). Chaos and Fractals: New Frontiers of Science. New York, Springer Verlag.

4. Alligood, K, Sauer, T et al (1997). Chaos: An Introduction to Dynamical Systems, Springer Verlag, New York,

Introductory Articles:

1. May, R. M. (1986). "When Two and Two Do Not Make Four."Proc. Royal Soc. B228: 241.
2. May, R.M. (1976) Simple mathematical models with very complicated dynamics. Nature, Lond. 261, 459-67 (1976).
3. Berry, M. V. (1981). "Regularity and Chaos in Classical Mechanics,Illustrated by Three Deformations of a Circular Billiard." Eur. J. Phys. 2: 91-102.
4. Crawford, J. D. (1991). "Introduction to Bifurcation Theory."Reviews of Modern Physics 63(4): 991-1038.
5. Shinbrot, T., C. Grebogi, et al. (1992). "Chaos in a Double Pendulum." Am. J. Phys 60: 491-499.
6. David Ruelle. (1980). "Strange Attractors," The Mathematical Intelligencer 2: 126-37.

Advanced Texts:

1. Arrowsmith, D. K. and C. M. Place (1990), An Introduction to Dynamical Systems.Cambridge, Cambridge University Press.
2. Guckenheimer, J. and P. Holmes (1983), Nonlinear Oscillations, Dynamical Systems, and Bifurcation of Vector Fields, Springer-Verlag New York.
3. Ott, E. (1993). Chaos in Dynamical Systems. Cambridge University Press,
4. Kattok et al., Introduction to Modern Dynamical Systems. Cambridge University Press.

Software:

UWO has a full site license for the Computer Algebra System Maple 10.
This software can be accessed using any PC or Mac at any computer lab on campus. Maple 10 is extremely user friendly and I expect you will be using it quite heavily to complete your homework and Maple assignments. I will give a general introduction to this software the first week of classes and shorter presentations on specific topics dealing with the extensive differential equations package included in Maple.

Website: I will to maintain a website for this course at http://www.uwosh.edu/faculty_staff/benzaid. The site will contain the syllabus, homework assignments, solutions to tests, solutions to homework problems, Maple 10 worksheets, miscellaneous lecture notes and links to other interesting Nonlinear Dynamics sites.

Exams and Grading:

Your grade will be based on two take home exams, 4 homework and Maple assignments, and 2 class presentations.

Exams:	45%
Homework and Maple Assignments:	40%
Presentations:	15%

Homework:

Homework problems from the book and Maple assignments will be assigned every week; they will be posted on the web. You are encouraged to work together and discuss problems with each other, but solutions must be worked out and submitted individually; you are responsible for your own homework. Please work neatly and clearly and explain your reasoning, and produce neat and clearly labeled graphs when appropriate.

Presentations:

The class will be divided into groups of 2 students. Each group will be responsible for giving two short presentations on an appropriate topic related to dynamical systems. Some possible presentation topics are:

- The mathematics and computation of fractals
- Further exploration of iterative maps: rigorous definitions and detailed proofs of chaos in one-dimensional maps, Sarkovskii's ordering and "period three implies chaos", universality; or numerical explorations of iterative maps, possibly in higher dimensions.
- Biological oscillations, models of population dynamics and ecology, epidemiology or immunology, models of HIV/AIDS dynamics
- Chemical or biochemical oscillations
- Physical applications, such as models for lasers
- Topics in classical mechanics
- Celestial mechanics, planetary motion, ...
- Pattern formation, such as convection in fluids or biological patterns
- Chaos and cryptography; controlling chaos
- Philosophical aspects: the implications of chaos for chance and determinism

- Other topics related to dynamics, bifurcation, chaos, complexity, ...

Course Objectives:

Upon successful completion of the course a student is expected to

- Analyze one and two dimensional flows graphically, analytically and numerically.
- Understand phase portraits
- Understand and compute fixed points and equilibrium solutions.
- Classify linear systems
- Study the stability of fixed points for nonlinear systems using linearization.
- Understand the concepts of stable and unstable manifolds.
- Analyze some one dimensional and two dimensional models arising in applications such as population growth, population dynamics such as predator-prey problems, nonlinear oscillators, linear and nonlinear circuits and have some appreciation of the range of physical and biological problems to which this theory is applicable.
- Understand the concept of a bifurcation and bifurcation diagrams and be familiar with the most common types of bifurcations.
- Understand the concept of limit cycle.
- Understand and be able to use the Poincare-Bendixson Theorem in simple cases.
- Use Liapunov functions in the study of stability and closed orbits.
- Understand and analyze some simple three dimensional flows graphically, analytically and numerically.
- Understand properties of important chaotic systems derived from nonlinear differential equations such as the Lorenz and Rossler systems.
- Gain a descriptive knowledge of strange attractors
- understand how chaotic attractors can be characterized by the Lyapunov exponent and by various types of dimension
- Gain a working knowledge of discrete chaotic dynamical systems.
- Understand periodic points and their role in chaotic systems.
- Be able to give the characteristic properties of discrete chaotic systems.
- Understand the mathematics of bifurcation.
- Understand the mathematics involved in the period doubling route to chaos.
- Able to describe the genealogy of periodic points.
- Understand the difference between chaotic behavior and random behavior.

- Use the computer algebra system Maple to compute solutions of dynamical systems, graph phase portraits and solutions, simulate nonlinear dynamical systems behavior, study and compute bifurcations points.