SUMMER 2007 67-717 PROBLEM SET 2

Reading Assignment: Chapter 4 of Strogatz. and start reading chapter 10.0 to 10.4. We will go through some of the following problems in class, others are due on Tuesday, June 26, 2007.

1. Problem 4.1.2
2. Problem 4.1.5
3. Problem 4.3.1
4. Problem 4.3.2
5. Problem 4.2.3
6. Problem 4.3.3
7. Problem 10.1.3
8. Problem 10.1.6
9. Problem 10.1.8
10. Problem 10.1.11
11. Problem 10.1.12
12. Problem 10.2.8
13. Problem 10.3.4
14. Problem 10.3.7
15. Problem 11.1.6
16. Problem 11.2.5
17. Problem 11.3.8
18. Find all fixed points and periodic points of period 2 for each of the given functions:

    (a) $F(x) = -x + 2$ (b) $F(x) = -2x - x^2$.

19. Describe the fate of the orbit of each of the following seeds under iteration of the function

    $$T(x) = \begin{cases} 2x, & \text{if } x < 1/2; \\ 2 - 2x, & \text{if } x \geq 1/2 \end{cases}$$

    (a) 2/3 (b) 1/6 (c) 2/5 (d) 1/8 (e) 1/4 (f) 1/2.

20. For each of the given functions, find all fixed points and determine whether they are attracting, repelling, or neutral

(a) $F(x) = (\pi/2)\sin x$ (b) $F(x) = 3x(1-x)$.

21. What can you say about fixed points for $F_c(x) = ce^x$ with $c > 0$? What does the graph of F_c tell you about these fixed points? Note that when $c = 1/e$, $F_c(1) = 1$.

22. Consider the function

$$T(x) = \begin{cases} 4x, & \text{if } x < 1/2; \\ 4 - 4x, & \text{if } x \geq 1/2 \end{cases}$$

Does T have any attracting cycles? Why or why not?

23. Each function undergoes a bifurcation of fixed points at the given parameter value. In each case use analytic or qualitative methods to identify this bifurcation as a tangent, pitchfork, or period doubling bifurcation or as none of these. Discuss the behavior of orbits near the fixed points in question at, before, and after the bifurcation.

(a) $F_\alpha(x) = x + x^2 + \alpha, \quad \alpha = 0$ (b) $F_\alpha(x) = \alpha \sin x, \quad \alpha = 1$.