CGI Workshop Manual

Using Native American Legends
to Teach Mathematics

Legends Retold by Students
Participating in the
Anishinabe Teachers for Anishinabe Children Pre-College Program

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NOTE: For a complete booklet of this mathematics resource with 14 additional legends and Ojibwe, Oneida, and Menominee number words included, contact Judith Hankes@uwosh.edu. Additional resources are available at http://www.mindsongmath.org/.

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Brother Bear

a true Menominee story retold by
Louise Bear and Terri Zhuckkahosee

A long time ago on the Menominee Indian Reservation there lived a very old couple. The husband loved hunting and fishing. One winter night he decided to go on a hunting trip. So the next day his wife packed some warm clothes and lots of dried meat and berries for him and he set off on his trip.

He followed deer trails through the woods for a long time without seeing a deer. Then, at the edge of a meadow, he spotted one. Carefully, he aimed the arrow in his bow and let it fly. The arrow struck the deer but only wounded it. The deer leaped into a thicket of trees, and the old man quickly followed. He tracked the deer for many miles but eventually lost its trail. When he finally decided to return home, he realized that he was lost. The old man panicked and started to run, but he could not find a familiar trail. The old man remained lost for many days.

Late one afternoon, while trying to save time, he decided to walk across a lake. He had not gone far, when suddenly he broke through a weak spot in the ice! He carefully pulled himself from the water and crawled to shore. There, he took off his snowshoes and other heavy clothing and started walking. He became very cold and tired. Fortunately, he noticed a small cave and went inside. A bear was hibernating in the cave, but that didn't bother him. The old man cuddled close to the bear to keep warm, and while lying next to the bear, he ate the dried meat and berries his wife had sent with him. He stayed with the bear until his clothing dried and he regained his strength.

When he finally returned home, his wife asked, “Where have you been?”

The old man answered, “I was staying with my brother bear.”

Brother Bear Word Problems

Join: Result Unknown

The old man lay next to the bear eating berries. First he ate ___ berries.
Then he ate ___ more. How many berries did the old man eat?

Separate: Result Unknown

The old man had ___ berries. He ate ___ of them.
How many berries didn’t he eat?

Part Part Whole: Whole Unknown

The old man has ___ blueberries and ___ strawberries. How many berries does the old man have?

Compare: Difference Unknown

The old man saw ___ crows and ___ blue jays.
How many more blue jays than crows did the old man see?

Multiplication

There were ___ oak trees. In each oak tree there were ___ blue jays. How many blue jays were there altogether

Measurement Division

The old man gave ___ berries to some animals. He gave ___ berries to each animal. How many animals got berries?

Two-Step Problem

To celebrate the old man’s safe return, the old lady held a feast. ___ friends came to the feast. Each friend ate ____ pieces of fry bread. There were ___ pieces of fry bread leftover. How many pieces of fry bread had the old woman fried?

PROBLEM-SOLVING SITUATIONS

JOINING PROBLEMS

Join: Result Unknown (JRU)	**Join: Change Unknown (JCU)**	**Join: Start Unknown (JSU)**
♦ Grandmother had 5 strawberries. Grandfather gave her 8 more strawberries. How many strawberries does Grandmother have now? 5 + 8 = □	♥ Grandmother had 5 strawberries. Grandfather gave her some more. Then Grandmother had 13 strawberries. How many strawberries did Grandfather give Grandmother? 5 + □ = 13	♠ Grandmother had some strawberries, Grandfather gave her 8 more. Then she had 13 strawberries. How many strawberries did Grandmother have before Grandfather gave her any? □ + 8 = 13

SEPARATING PROBLEMS

Separate: Result Unknown (SRU)	**Separate: Change Unknown (SCU)**	**Separate: Start Unknown (SSU)**
♦ Grandfather had 13 strawberries. He gave 5 strawberries to Grandmother. How many strawberries does Grandfather have left? 13 - 5 = □	♥,Grandfather had 13 strawberries. He gave some to Grandmother. Now he has 5 strawberries left. How many strawberries did Grandfather give Grandmother? 13 - □ = 5	♠ Grandfather had some strawberries. He gave 5 to Grandmother. Now he has 8 strawberries left. How many strawberries did Grandfather have before he gave any to Grandmother □ - 5 = 8

PART -PART -WHOLE PROBLEMS

Part-Part-Whole: Whole Unknown (PPW:WU)	**Part-Part-Whole: Part Unknow (PPW:PU)**
♦ Grandmother has 5 big strawberries and 8 small strawberries. How many strawberries does Grandmother have altogether? 5 + 8 = □	♥,Grandmother has 13 strawberries. Five are big and the rest are small. How many small strawberries does Grandmother have? 13 - 5 = □ or 5 + □ = 13

<u>COMPARE PROBLEMS</u>

Comp. Difference Unknown	**Comp. Quantity Unknown**	**Comp. Referent Unknown**
♦♥,Grandfather has 8 strawberries. Grandmother has 5 strawberries. How many more berries does Grandfather have than Grandmother? 8 - 5 = □ or 5 + □ = 8	♠ Grandmother has 5 strawberries. Grandfather has 3 more strawberries than Grandmother. How many strawberries does Grandfather have? 5 + 3 = □	♠ Grandfather has 8 strawberries. He has 3 more strawberries than Grandmother. How many strawberries does Grandmother have? 8- 3 = □ or □ + 3 = 8

MULTIPLICATION & DIVISION PROBLEMS

Multiplication	**Measurement Division**	**Partitive Division**
♦ Grandmother has 4 piles of strawberries. There are 3 strawberries in each pile. How many strawberries does Grandmother have? 4 x 3 = □	♦ Grandmother had 12 strawberries. She gave them to some children. She gave each child 3 strawberries. How many children were given strawberries? 12 ÷3 = □	♦♥,Grandfather has 12 strawberries. He wants to give them to 3 children. If he gives the same number of strawberries to each child, how many strawberries will each child get? 12 ÷ 3 = □

Problem chart based on Cognitively Guided Instruction Problem Types (Carpenter et al., 1996)

UNDERSTANDING THE STRUCTURE OF WORD PROBLEM
What makes a problem easy or difficult?

A goal of Cognitively Guided Instruction is that young children become independent problem solvers who are able to approach and solve word problems without having to rely on having a teacher tell them how to do it. However, a number of factors influence whether a problem is appropriate for a child to solve independently. Understanding these factors helps the teacher decide which word problems to use during instruction. These factors include the following:

If the Problem Involves a Situation That the Child Can Act Out

A problem that can be acted out is easier for a child to solve than one that cannot be acted out. For example, the first of the following two problems is easier. Here the child can actually pretend that s/he is giving strawberries away. The second problem is more difficult because it requires more thought to make sense of the question being asked.

SRU (Action Direct):
Grandfather had 8 strawberries. He gave 3 of them to Grandmother. How many strawberries does Grandfather have now?

SRU (Action Indirect):
Grandfather gave 3 strawberries to Grandmother. He had 8 strawberries. How many strawberries does Grandfather have now?

If the Child is Able to Model the Problem
with Counters or Drawing

When the quantities given in a problem refer to a complete set of physical objects or amounts, the problem can be modeled directly. When a word problem can be directly modeled, that is, represented in some concrete way on fingers, with tally marks, drawings, or by manipulating counters, the problem is easier. The first of the following two problems is easier because the wording guides the child's modeling. When modeling this problem with counters, a young child might choose to set out the two quantities, lining them up side by side, and then match them to determine the difference. Solving the second problem relies on the child's ability to mentally determine the relationship between quantities within the problem.

CDU direct modeling situation:
Grandfather has 8 strawberries. Grandmother has 5 strawberries. How many more strawberries does Grandfather have?

CRU situation that requires ability to analyze:
Grandmother has 5 strawberries. She has 3 fewer strawberries than Grandfather. How many strawberries does Grandfather have?

Multiplication and division problems can be modeled. Therefore, young children can multiply and divide intuitively

Very young children can solve low number multiplication and division problems because such problems can be easily modeled. However, it is important that children first solve many problems involving joining and separating situations. These experiences will allow them to develop the ability to think about numerical quantities within the context of words and to make sense of the question being asked.

Multiplication problem:
Grandmother has 4 piles of strawberries. There are 3 strawberries in each pile. How many strawberries does Grandmother have?

Measurement Division problem:
Grandmother gave 12 strawberries to some children. She gave each child 3 strawberries. How many children got strawberries?

Partitive Division problem:
If Grandfather shares12 strawberries with 3 friends, how many strawberries will each friend get?

If a problem can be modeled or acted out in the order in which it is heard, it is easier.

When first learning to solve word problems, young children approach them in the order in which they hear them. They do not begin at the end of the problem and work backward. The ability to use inverse thinking develops after children have had many experiences with solving problems and have developed an understanding of the relationships among the numbers within a problem - they understand that combined parts make up the total. For this reason, the first of the following two problems is easier. The wording encourages the child to set out five counters and then add eight more. However, the second problem does not provide a beginning number. The child who has not yet developed the ability to relate a part of a quantity to the total quantity will respond to the second question with, "Some. Grandmother had some strawberries."

Joining problem that can be solved in the word order given:
Grandmother had 5 strawberries. Grandfather gave her 8 more.
How many strawberries does Grandmother have now?

Joining problem that cannot be solved in the order given:
Grandmother had some strawberries. Grandfather gave her 8 more.
Then she had 13 strawberries.
How many strawberries did Grandmother have before Grandfather gave her any?

The Location of the Unknown Influences the Problem Difficulty

Because young children solve problems in the order that they hear them, problems that are worded in such a way so that the unknown quantity is located at the end (first example below) are easier to solve. Problems with the missing quantity in the middle (second example below) or at the beginning (third example below) are more difficult.

As the child's understanding of quantity and relationships among quantities develops, s/he becomes able to make sense of the entire question, represent the situation, and plan a solution. When a child is able to do these steps s/he will not need to use manipulatives. Rather, the child will use his or her own unique way of mentally manipulating quantities.

SRU location of unknown at end of problem: (8 - 3 = __)
Grandmother had 8 strawberries. She gave 3 to Grandfather. How many strawberries does Grandmother have now?

SCU location of unknown in middle of problem: (8 - __ = 5)
Grandfather had 8 strawberries. He gave some to Grandmother. Now he has 5 strawberries. How many strawberries did Grandfather give to Grandmother?

SSU location of unknown at start of problem: (__ - 3 = 5)
Grandfather had some strawberries. He gave 3 strawberries to Grandmother. Then he had 5 strawberries left. How many strawberries did Grandfather have before sharing with Grandmother?

Children's Intuitive Solution Strategies

Extensive research has documented the developmental thinking processes that children go through when learning to solve word problems (Carpenter et al., 1992). It is important to emphasize that these processes are **intuitive**, ones that are not taught to the student by a teacher.

To effectively promote the development of mathematical reasoning without usurping the student's intuitive thinking, a teacher must clearly understand the relationships among the different types of word problems (discussed in the previous sections) and the developmental stages of children's thinking. Detailed descriptions of how children's solutions vary depending on their developmental ability are provided in the following sections.

Relating Solution Strategies to the Developmental Stages of Mathematical Reasoning

The following word problems are used to demonstrate how children at different developmental levels will use different strategies when solving the same problems. The strategy that the child uses indicates the child's stage or level of development.

Join: Result Unknown (JRU)

Grandfather had 6 strawberries. Grandmother gave him 5 more. How many strawberries does Grandfather have now?

Separating: Result Unknown (SRU)

Grandmother had 11 strawberries. She gave 5 to Grandfather. How many strawberries does Grandmother have now?

Developmental Level I
Direct Modeling

A child using a **Direct Modeling** strategy represents each number in the problem with concrete objects. In the following examples, the child solves the Join Result Unknown (JRU) and the Separate Result Unknown (SRU) problem given above by modeling with counters.

Child's Solution to JRU

"Grandfather had 6 strawberries.
strawberries.
One, two, three, four, five, six."
(The child sets our 6 counters.)
"Grandmother gave him five
More. One, two, four, five."
(Child sets out 5 counters and then pushes both sets together and counts all of the counters.)
"Now he has 11 strawberries."

Child's Solution to SRU

"Grandmother had 11
One, two, three, four, five, six, seven
eight, nine, ten, eleven."
(*Child sets out 11 counters.)*
"She gave 5 to Grandfather. One, two, Three, four, five." *(Child counts out and removes 5 counters from the group of 11 and counts the remaining counters.)* "Now she has. . . one, two, three, four, five, six. She has six."

Developmental Level II
Counting On/Back

A child using a **Counting On/Back** strategy is able to hold a number in her/his mind and count on or back from that number while keeping track of the quantity that is added or subtracted using fingers, tally marks, or counters.

A child at this level is able to immediately recognize groups such as the amount modeled on his/her fingers without having to recount the fingers. In the following examples the child solves the Join Result Unknown (JRU) and Separate Result Unknown (SRU) problems (problems given above)) using counting strategies.

Child's Solution to JRU

"I don't have to count the six again. I just have to add five to it. I say, 'Seven, eight, nine, ten, eleven'." *(Child holds up a finger with each count.)* "I have eleven."

Child's Solution to SRU

"I know Grandmother had eleven strawberries. I know she gave five away. So, I count five down. 'Eleven, ten, nine, eight, seven.' I have six left." (*Child folds a finger down with each count.)*

Developmental Level III
Deriving

A child possessing good number sense is able to solve problems in flexible ways, often breaking numbers down and recombining them by using known facts. This child frequently visualizes the quantities and solves the problem with mental math.

Child's Solution to JRU (Above)

"I know that five and five is ten. I took one from the six to make five. But I must add the one back on. It's eleven."

Child's Solution to SRU (Above)

"I know that ten take away five is five, but I started with eleven. The answer must be one more. It's six."

Matching Solution Strategies to Problem Types

The problem solving decisions that a child makes when solving a problem are determined by the problem situation posed to the child. Each of the 14 different problem types requires different reasoning processes. Examples of these processes at each developmental level are provided in this section.

Level I
Direct Modeling Strategies

At the **Direct Modeling** level, the child concretely represents (using counters, fingers, tally marks, drawings) all numerical quantities within the problem. Below are examples of problems with direct modeling solutions.

JRU Problem

Grandfather had 3 strawberries. Grandmother gave him 5 more. How many strawberries does Grandfather have now?

Solution: Joins-All
The child constructs (with manipulatives or drawing) a set of three objects and a set of five objects. The child pushes the sets together and the union of the two combined sets is counted.

JCU Problem

Grandmother has 5 strawberries. Grandfather gave her some more strawberries.
Now Grandmother has 8 strawberries.
How many strawberries did Grandfather give her?

Solution: Joins-To
The child constructs (with manipulatives or drawing) a set of three objects.
Objects are added to this set until there is a total of eight objects.
The child counts the number of objects that were added to find the answer.

SRU Problem

Grandfather had 8 strawberries. He gave 3 to Grandmother.
How many strawberries does Grandfather have now?

Solution: Separates-From
The child constructs (with manipilatives or drawings) a set of a set of eight objects. Three objects are removed. The answer is the number of remaining objects.

SCU Problem

Grandmother had 8 strawberries. She gave some to Grandfather.

Now Grandmother has 3 strawberries.
How many strawberries did she give to Grandfather?

Solution: Joins-To
A set of eight objects is counted out. Objects are removed from the set until the number of objects remaining is equal to three. The answer is the number of objects removed.

CDU Problem
Grandfather has 3 strawberries. Grandmother has 5 strawberries.
How many more strawberries does Grandmother have than Grandfather?

Solution: Matches
A set of three objects and a set of eight objects are matched one to one until one set is used up. The answer is the number of unmatched objects remaining in the larger set.

JSU Problem
Grandmother had some strawberries. Grandfather gave him 3 more.
Now he has 8 strawberries.
How many strawberries did Grandmother have to start with?

Solution: Trial-and-Error
A set of objects is constructed. A set of three objects is added to or removed, and the resulting set is counted. If the final count is eight, then the number of objects in the initial set is the answer. If it is not right, then a different initial set is tried.

Level II
Counting On/Back Strategies

At the **Counting On/Back** level, the child does not have to represent all quantities in the problem concretely. S/he has learned that a number names a quantity, that is, that a number can be stated rather than represented concretely. These strategies will develop intuitively over time. If a child is not able to make sense of counting strategies, the child is not developmentally ready to use them and needs more experience modeling solutions.

JRU Problem
Grandfather had 3 strawberries. Grandmother gave him 5 more strawberries.
How many strawberries does Grandfather have now?

Solution: Counts-On-From-First Number Heard
The child begins with 3 (the first number in the problem and continues on for 5 more counts (keeping track of counts with manipulatives, tallies, or fingers). The answer is the last number in the counting sequence.

JRU Problem
Grandfather had 3 strawberries. Grandmother gave him 5 more strawberries.

How many strawberries does Grandfather have now?

Solution: Counts-On-From-Larger
The child begins with 5 (the larger quantity and continues on for 3 more counts (keeping track of counts with manipulatives, tallies, or fingers. The answer is the last number in the counting sequence.

JCU Problem
Grandmother had 3 strawberries. Grandfather gave her some more strawberries. Now Grandmother has 8 strawberries.
How many strawberries did Grandfather give her?

Solution: Counts-On-To
The child uses a forward counting sequence starting from 3 and continues until 8 is reached (keeping track on fingers or tally marks). The answer is the number of counting words in the sequence.

SRU Problem
Grandfather had 8 strawberries. He gave 3 to Grandmother.
How many strawberries does Grandfather have now?

Solution: Counts-Down
The child uses a backward counting sequence starting from eight. The sequence continues for three counts . . . eight, seven, six . . . the answer is the next number.

SCU Problem
Grandmother had 8 strawberries. She gave some to Grandfather.
Now she has 3 strawberries. How many strawberries did she give to Grandfather?

Solution: Counts Down To
The child uses a backward counting sequence starting with 8 and continues until 3 is reached, "8, 7, 6, 5, 4 . . . " The answer is the total of number words in the counting sequence, not including the number word three.

Level III
Deriving Strategies

At this level, the child understands relationships between numbers. S/he solves problems using number facts and derived facts (combines familiar quantities when a specific fact is not at the recall level). An example of a child using a derived fact would be, "I know that nine plus four is thirteen because nine and one is ten and three more is thirteen."

Solution Strategies Summary

When children begin to solve problems intuitively, they concretely represent the relationships in the problem. Over time, concrete strategies are abstracted to counting strategies, and eventually, as number facts are learned, children apply this knowledge to solve problems. This developmental approach differs from the practice of rote drill for memorization of facts. Children in drill/skill classrooms often are able to recite facts but lack understanding that a fact represents a relationship between quantities; they lack mathematical reasoning in relation to number sense. Children who have been allowed to progress through the stages of mathematical reasoning described in this manual develop both number sense and mathematical reasoning.

SYMBOLIC PROCEDURES

Much of what has been discussed to this point has focused on children's informal or intuitive problem-solving strategies. Such strategies are often very different from the standard symbolic procedures typically taught in the elementary school. Standard procedures provide powerful problem-solving tools; however, a concern is that many children merely memorize them. They never develop an understanding of the relationships among numbers within procedures. When allowed to progress through the stages described in the preceding section, a child will develop the habit of looking for numerical relationships. When introduced to the standard procedure, this child will understand the numerical relationships and will view the procedure simply as another strategy for solving problems.

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Resource Information

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A text describing CGI, Children's Mathematics: Cognitively Guided Instruction, is available through the Heinemann web site: http://www.heinemann.com

CGI web sites:

Blog: http://mindsongmath.blogspot.com/

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http://www.abacon.com/ie/berk/wlp452a.htm